Principles of Communications

ECS 332

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7. Pulse Modulation, ISI, and Pulse Shaping

Office Hours:

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Tuesday	9:00-10:00
Wednesday	14:20-15:20
Thursday	9:00-10:00

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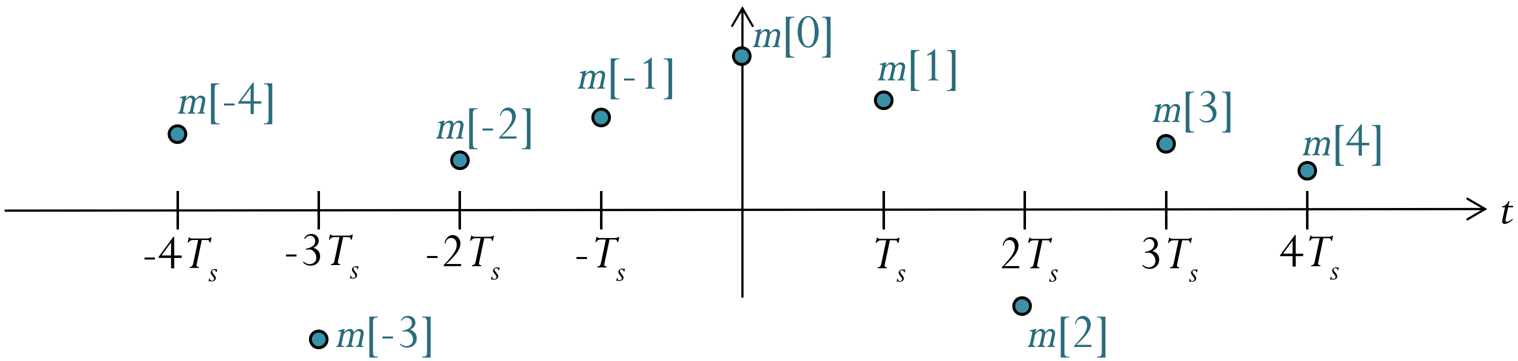
7.1 Analog Pulse Modulation

PAM: Pulse Amplitude Modulation

Start with a sequence of symbols (numbers).



Where does this sequence come from?

- Sampling of a continuous-time signal
- Naturally discrete-time signal

Naturally digital information

- An ordered sequence of **symbols** (or characters)
- Produced by a discrete information source.
- The source draws from an **alphabet** of $M \geq 2$ different symbols.
  - Ex. English text source: 26 (a to z) + 26 (A to Z) + 10 (0 to 9) + Punctuation and Other Signs (. , ! @ ( ))
  - Ex. Thai text source: 44 consonants (พยัญชนะ) + 15 vowel symbols (สระ) + 4 tone marks (วรรณยุกต์) + …
  - Ex. A typical computer terminal has an alphabet of $M \approx 90$ symbols (the number of character keys multiplied by two to account for the shift key)

Naturally digital information

- Text is commonly encoded using ASCII, and MATLAB automatically represents any string file as a list of ASCII numbers.

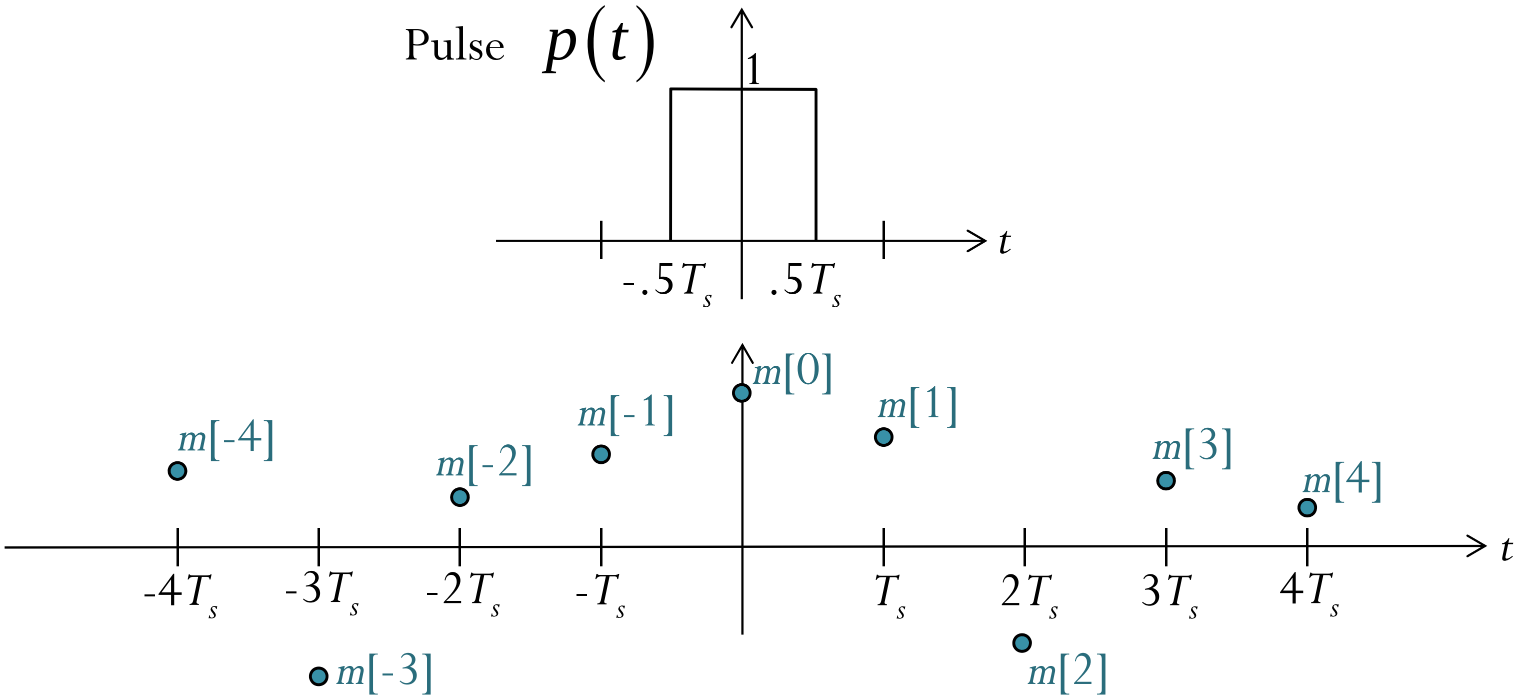
```
>> str='I love ECS332';
>> real(str)

ans =

    73    32   108   111   118   101    32    69    67    83    51    51    50

>> dec2base(str,2)

ans =

1001001
0100000
1101100
1101111
1110110
1100101
0100000
1000101
1000011
1010011
0110011
0110011
0110010
```

text string

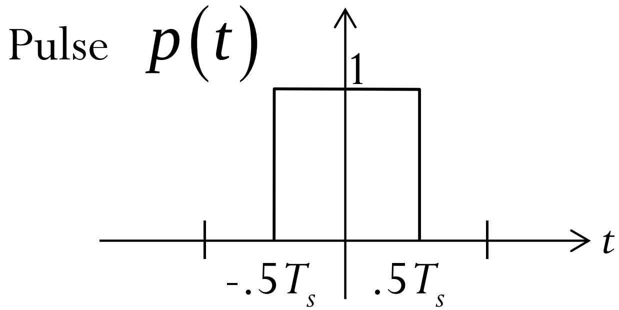
(decimal) ASCII representation of the text string

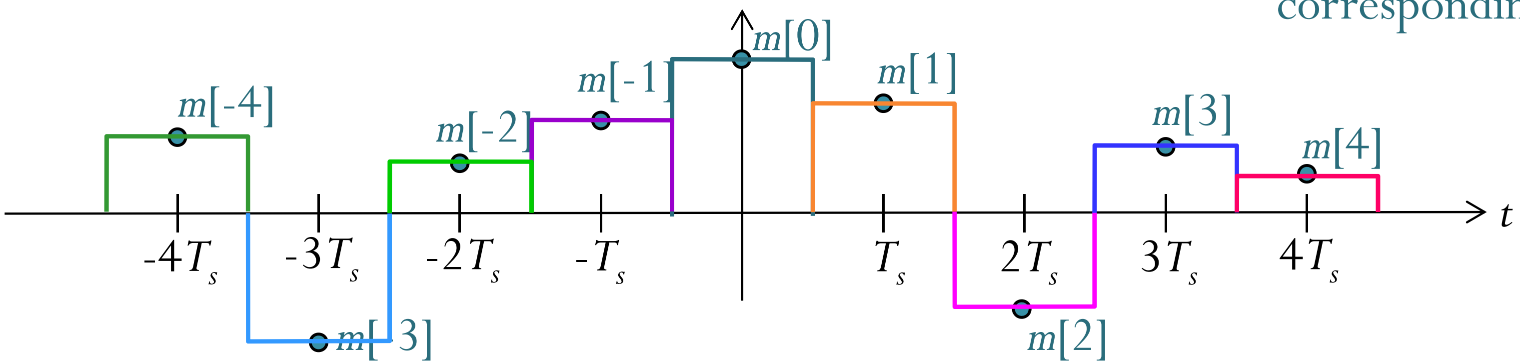
binary (base 2) representation of the decimal numbers

PAM: Example 1



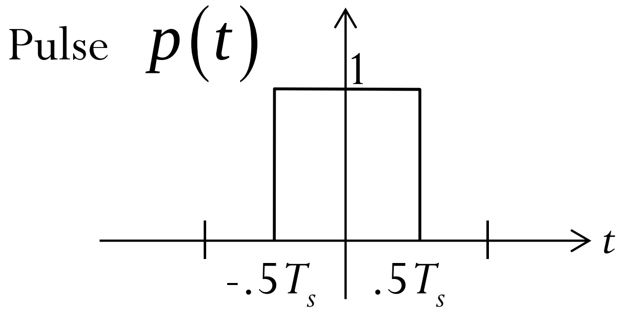
PAM: Example 1

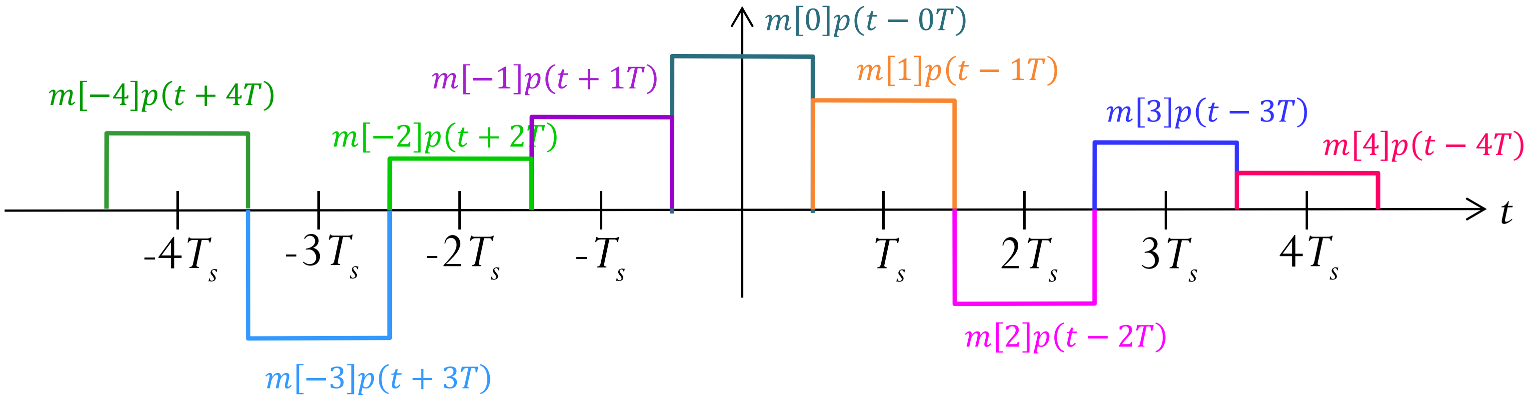




The height (amplitude) of each pulse is scaled by the corresponding $m[n]$

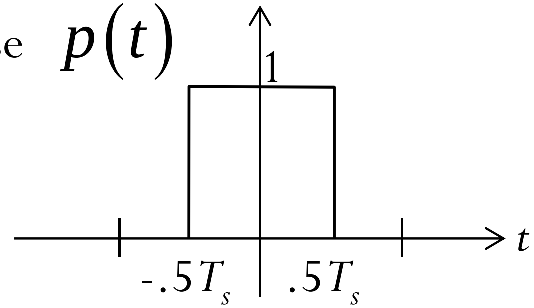
PAM: Example 1

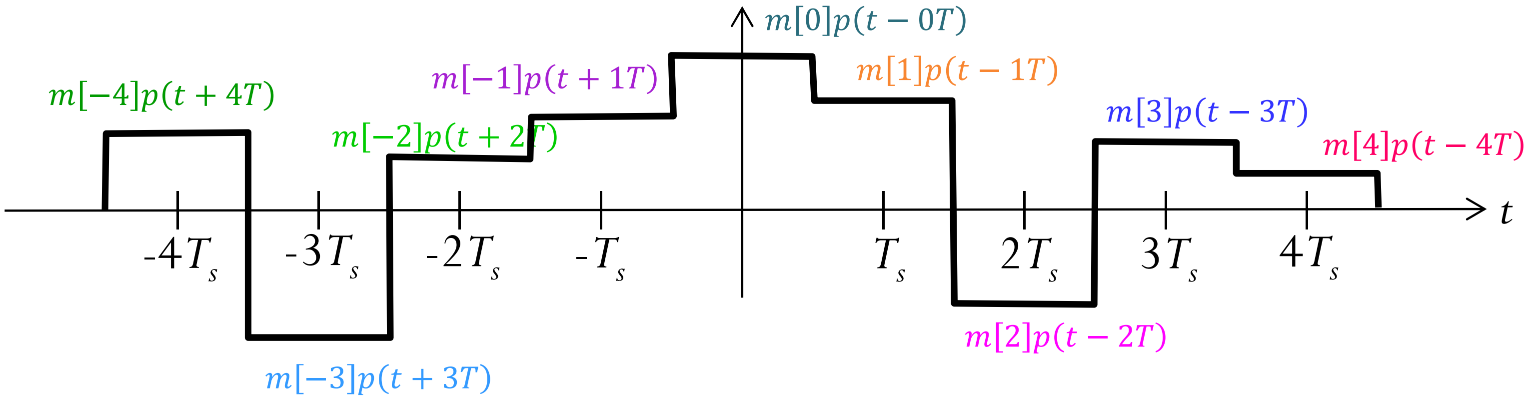




$$x_{\mathrm{PAM}}(t)=\sum_{n=-\infty}^{\infty} m[n]\,p(t-nT)$$

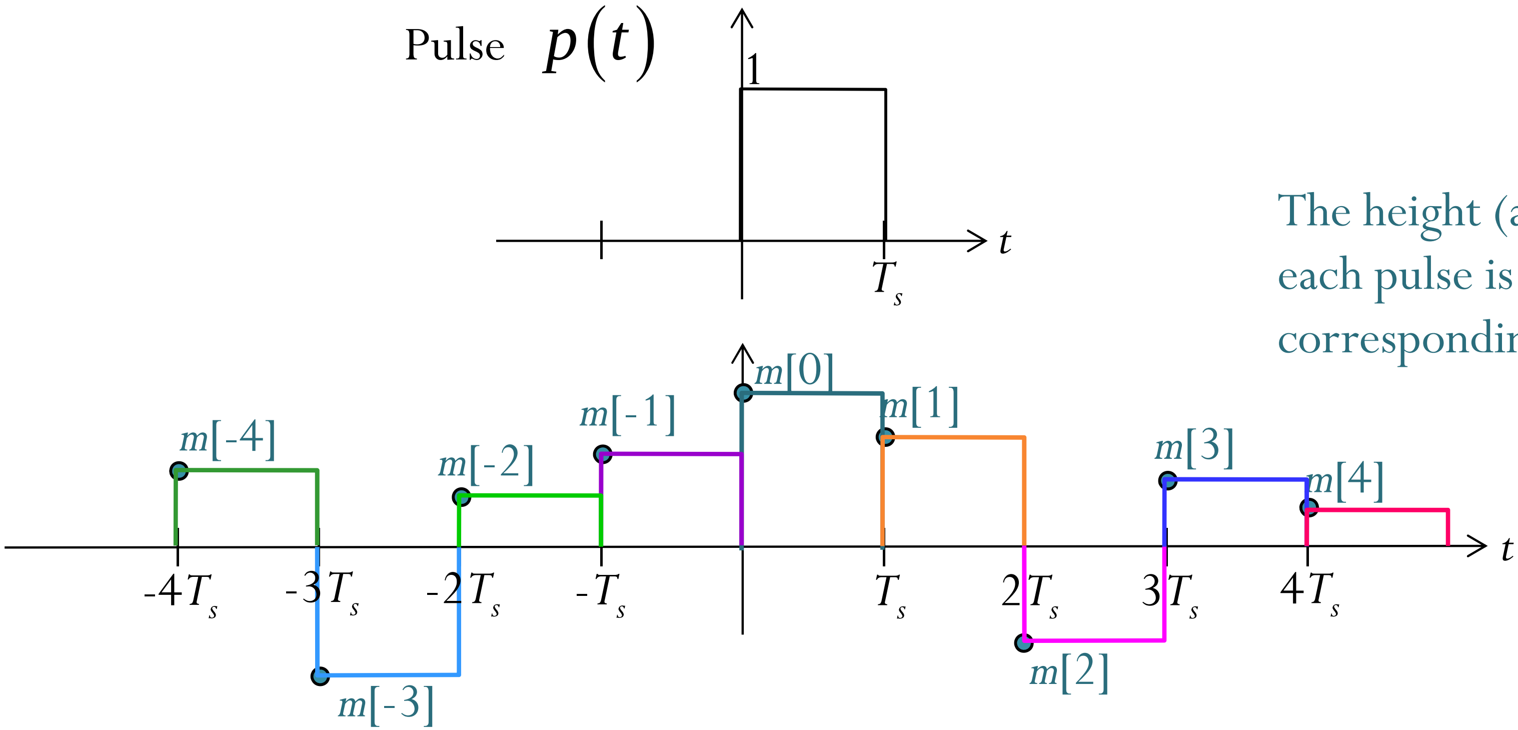
PAM: Example 1





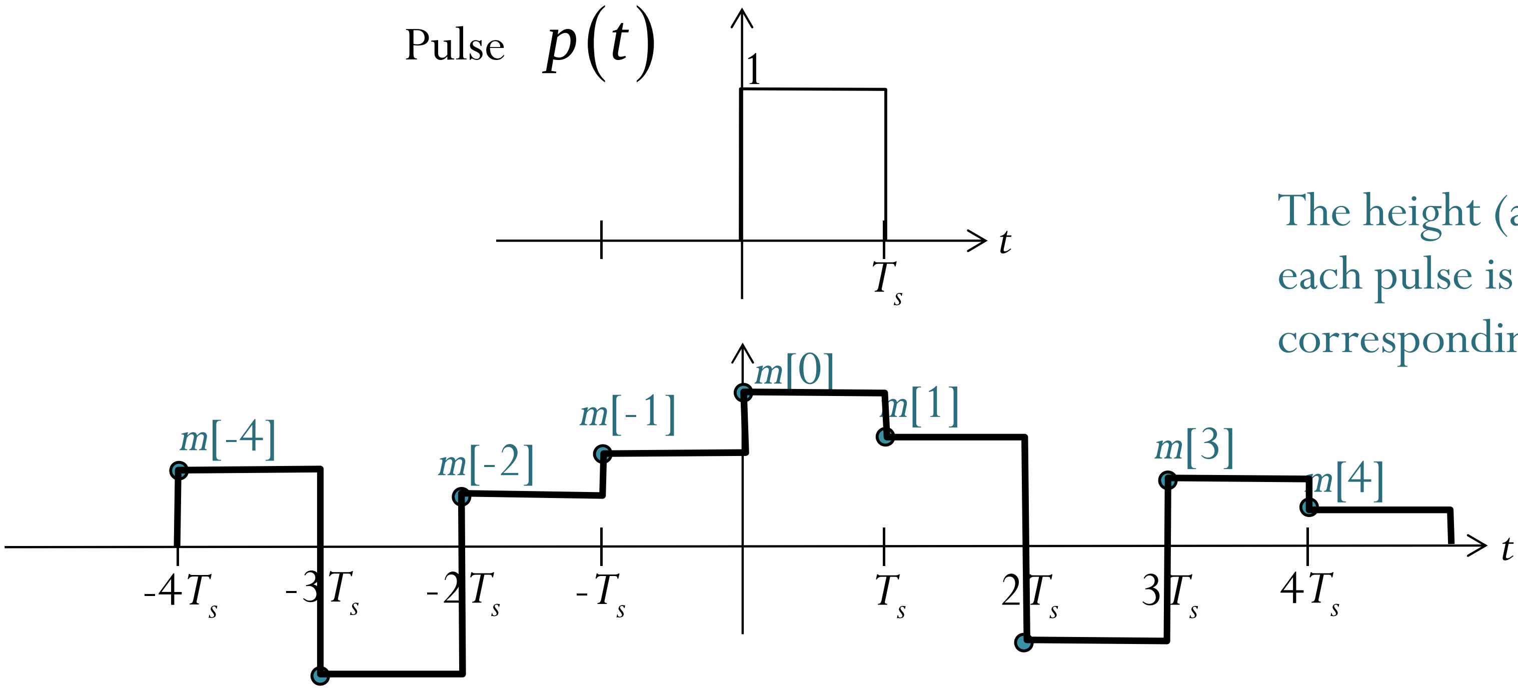
$$x_{\text{PAM}}(t) = \sum_{n=-\infty}^{\infty} m[n]\, p(t - nT)$$

PAM: Example 2



The height (amplitude) of each pulse is scaled by the corresponding $m[n]$

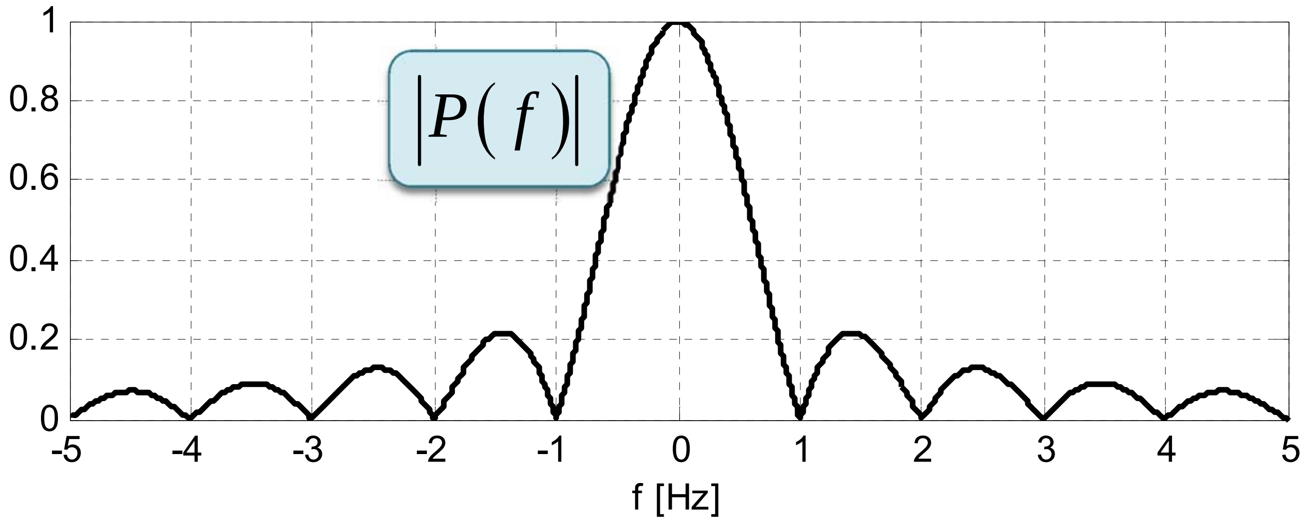
PAM: Pulse Amplitude Modulation

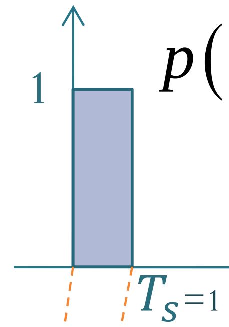


The height (amplitude) of each pulse is scaled by the corresponding $m[n]$

$$x_{\text{PAM}}(t) = \sum_{n=-\infty}^{\infty} m[n]\, p(t - nT)$$

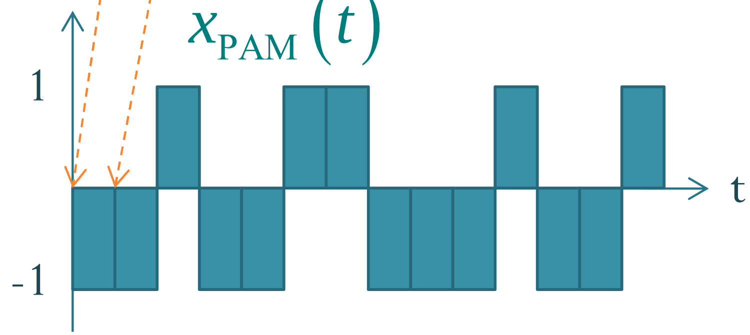
$X_{\mathrm{PAM}}(f)$ (1/4)





$$p(t) = 1\left[t \in [0, T_s)\right]$$

m= [-1,-1,1,-1,-1,1,1,-1,-1,-1,1,-1,-1,1,-1,1,1,-1,-1,-1,-1,1,-1,-1,-1,-1,-1,1,-1,1]

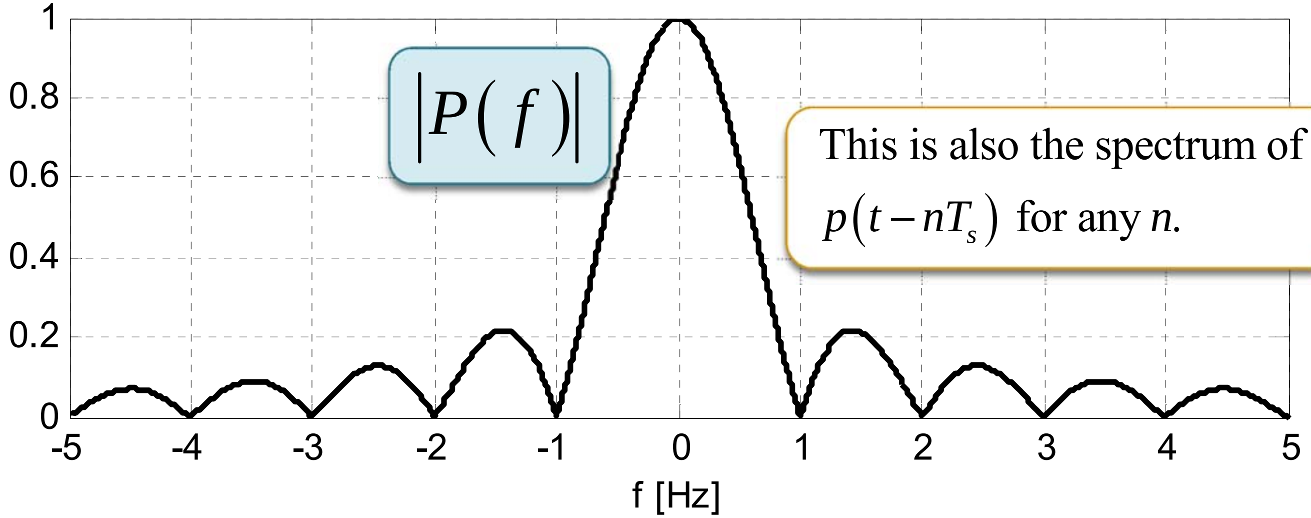


$$x_{\mathrm{PAM}}(t)$$

$$x_{\mathrm{PAM}}(t) = \sum_n m[n]\, p(t - nT_s)$$

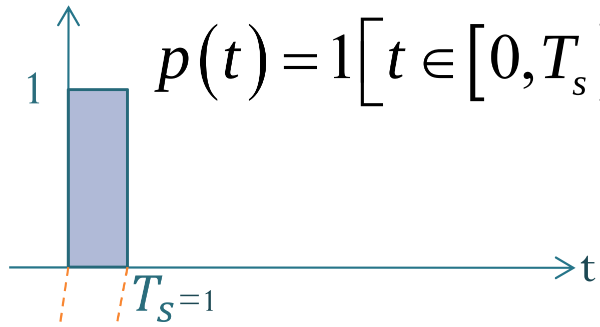
Can you sketch the spectrum of $s(t)$?

$X_{\mathrm{PAM}}(f)$ (2/4)

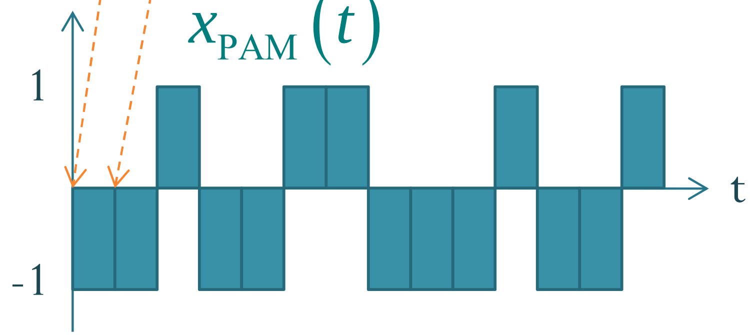


This is also the spectrum of $p(t-nT_s)$ for any n.

$p(t) = 1[t \in [0, T_s)]$



m= [-1,-1,1,-1,-1,1,1,-1,-1,-1,1,-1,-1,1,-1,1,1,-1,-1,-1,-1,1,-1,-1,-1,-1,-1,1,-1,1]



$$x_{\mathrm{PAM}}(t) = \sum_n m[n]\, p(t - nT_s)$$

Does this mean $|X_{\mathrm{PAM}}(f)|$ will simply be a sum of $|P(f)|$ and therefore its shape will be similar to $|P(f)|$?

Important Properties of $\mathcal{F}$

$$\{x * y\}(t) = \int_{-\infty}^{\infty} x(\mu)y(t-\mu)d\mu = \int_{-\infty}^{\infty} x(t-\mu)y(\mu)d\mu$$

Convolution Properties:

$$x * y \xrightleftharpoons{\mathcal{F}} X \times Y$$

$$x \times y \xrightleftharpoons{\mathcal{F}} X * Y$$

Note that the magnitude of this is simply $|G(f)|$

Shifting Properties:

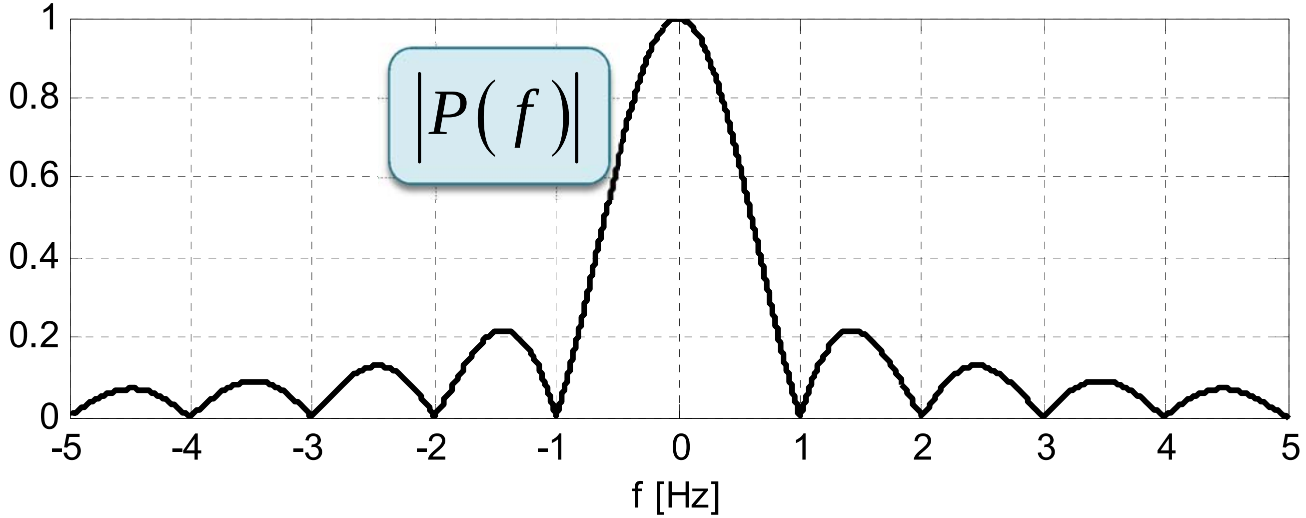
$$g(t - t_0) \xrightleftharpoons{\mathcal{F}} e^{-j2\pi f t_0} G(f)$$

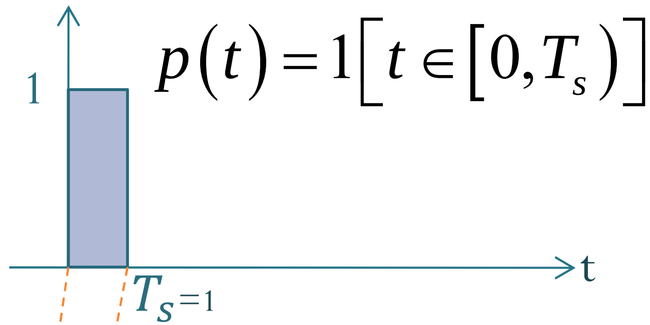
$$e^{j2\pi f_0 t} g(t) \xrightleftharpoons{\mathcal{F}} G(f - f_0)$$

Modulation:

$$g(t)\cos(2\pi f_c t) \xrightleftharpoons{\mathcal{F}} \frac{1}{2}G(f - f_c) + \frac{1}{2}G(f + f_c)$$

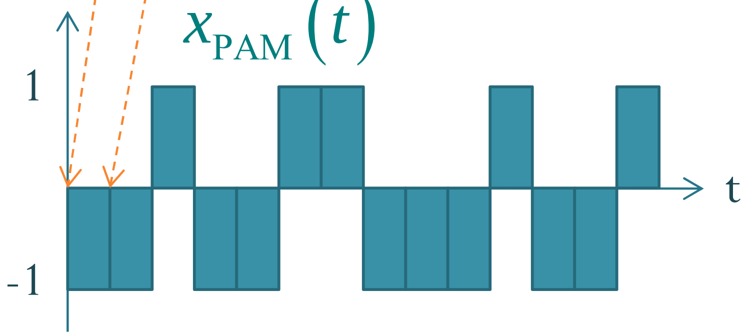
$X_{\mathrm{PAM}}(f)$ (3/4)





$$p(t) = 1\left[t \in [0, T_s)\right]$$

m= [-1,-1,1,-1,-1,1,1,-1,-1,-1,1,-1,-1,1,-1,1,1,-1,-1,-1,-1,1,-1,-1,-1,-1,-1,1,-1,1]

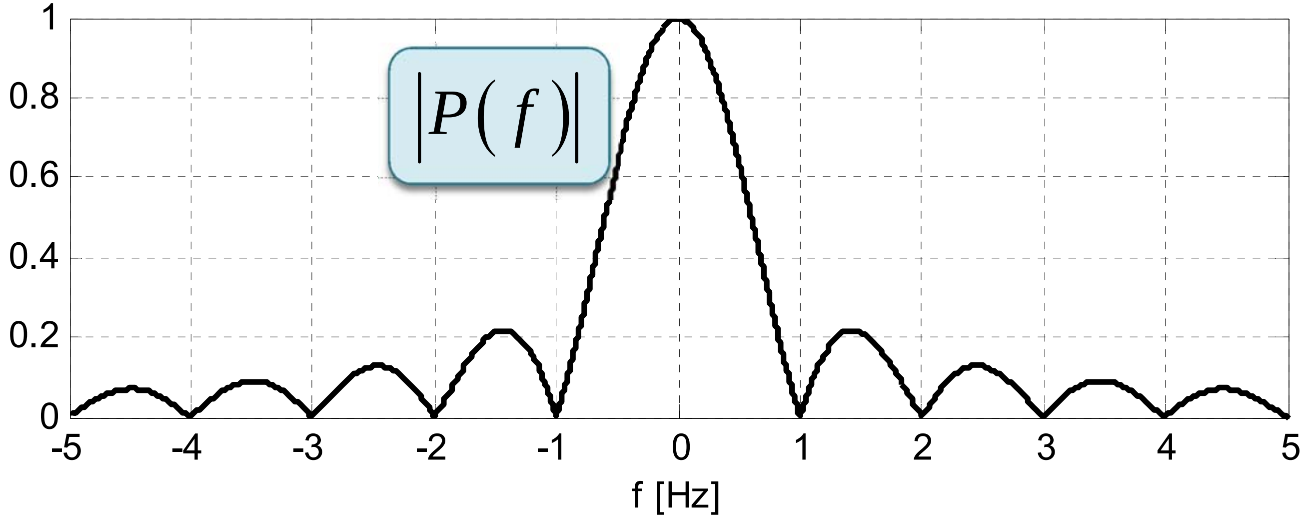


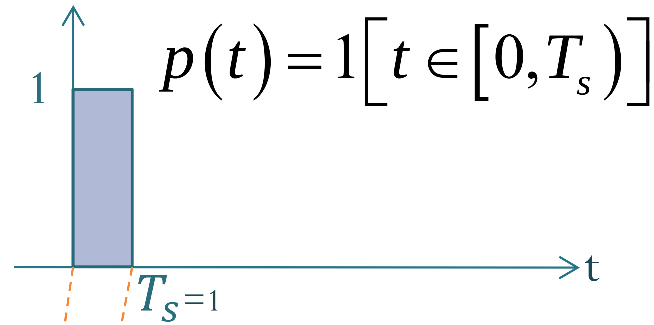
$$x_{\mathrm{PAM}}(t) = \sum_n m[n]\, p(t - nT_s)$$

$$\xrightarrow{\mathcal{F}} X_{\mathrm{PAM}}(f) = \sum_n m[n] P(f) e^{-j2\pi f n T_s}$$

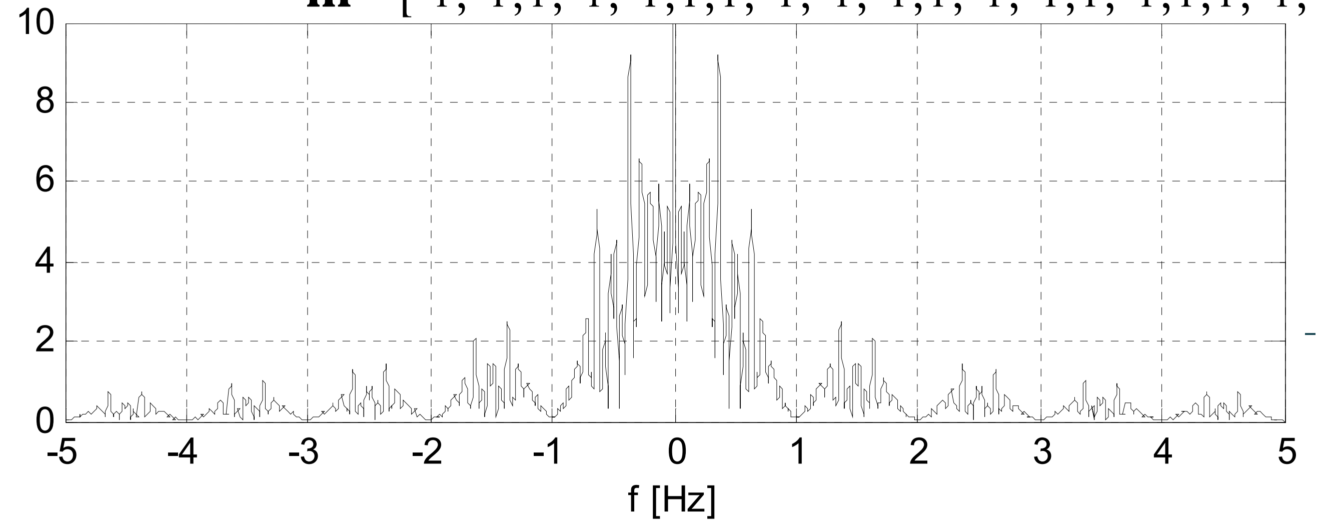
$$= P(f) \sum_n m[n] e^{-j2\pi f n T_s}$$

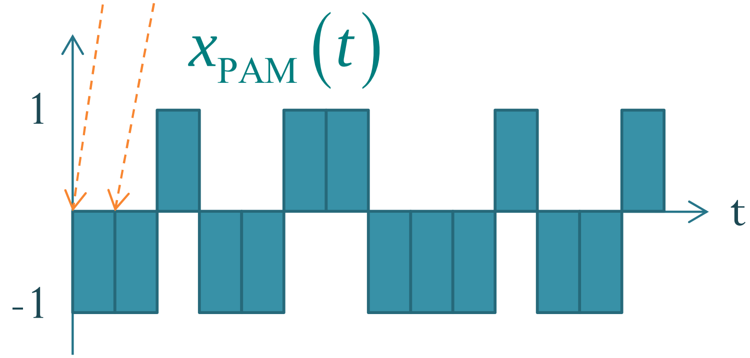
$X_{\mathrm{PAM}}(f)$ (4/4)





m= [-1,-1,1,-1,-1,1,1,-1,-1,-1,1,-1,-1,1,-1,1,1,-1,-1,-1,-1,1,-1,-1,-1,-1,-1,1,-1,1]

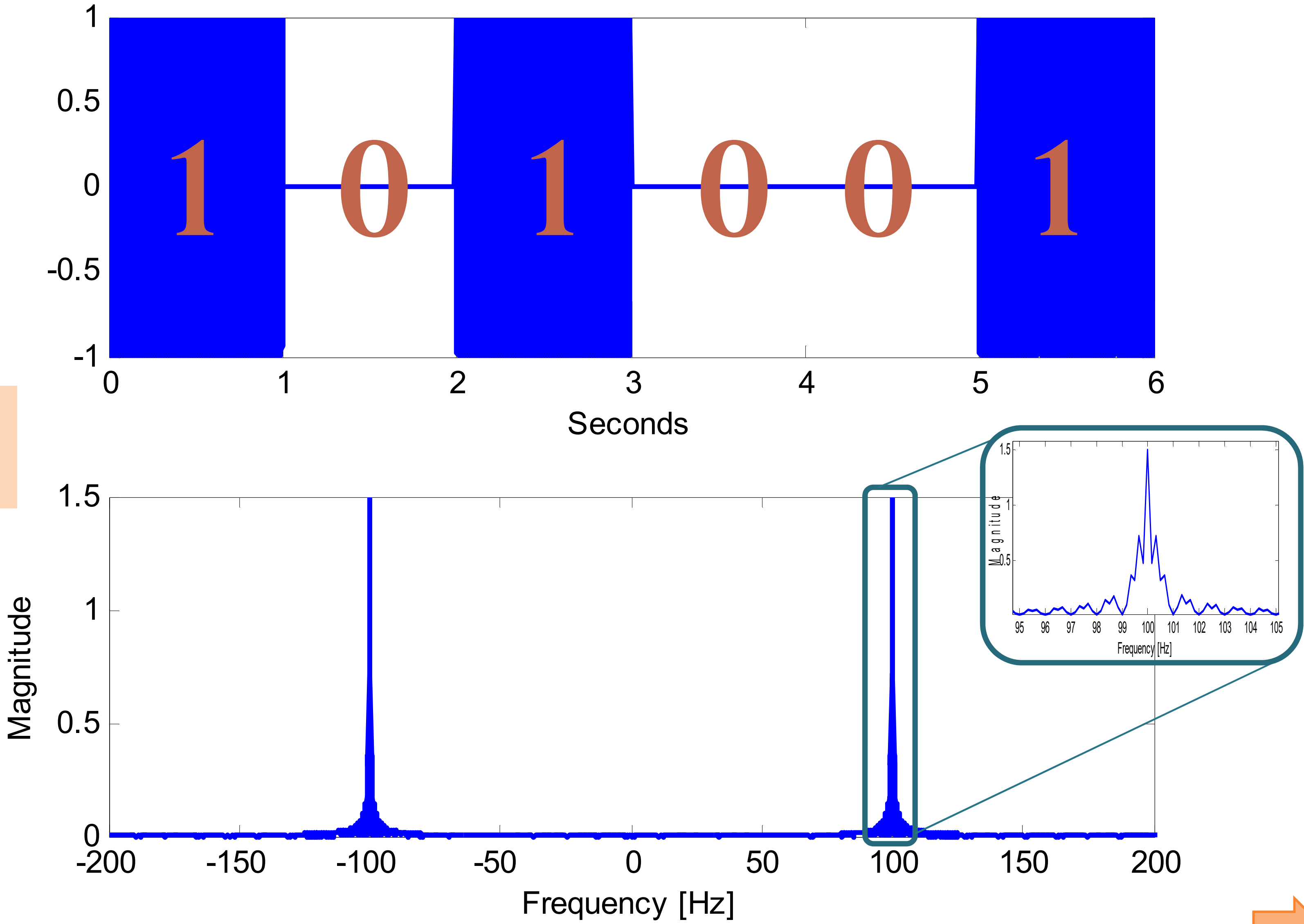




$$x_{\mathrm{PAM}}(t) = \sum_n m[n]\, p(t - nT_s) \xrightarrow{\mathcal{F}} X_{\mathrm{PAM}}(f) = P(f) \sum_n m[n] e^{-j2\pi f n T_s}$$

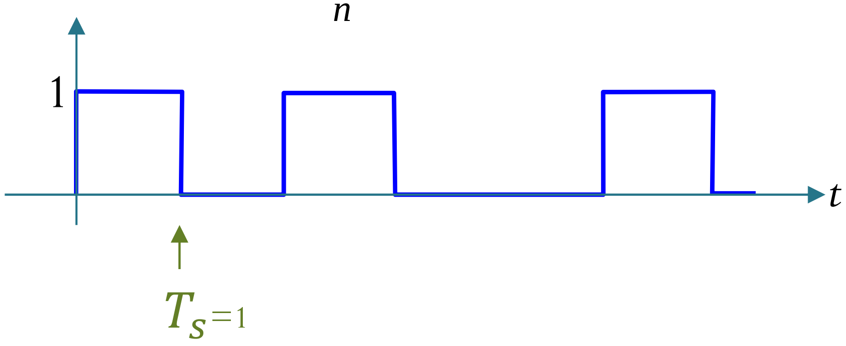
A revisit to an earlier OOK Example

$f_c = 100$ Hz
Bit rate = 1 bps

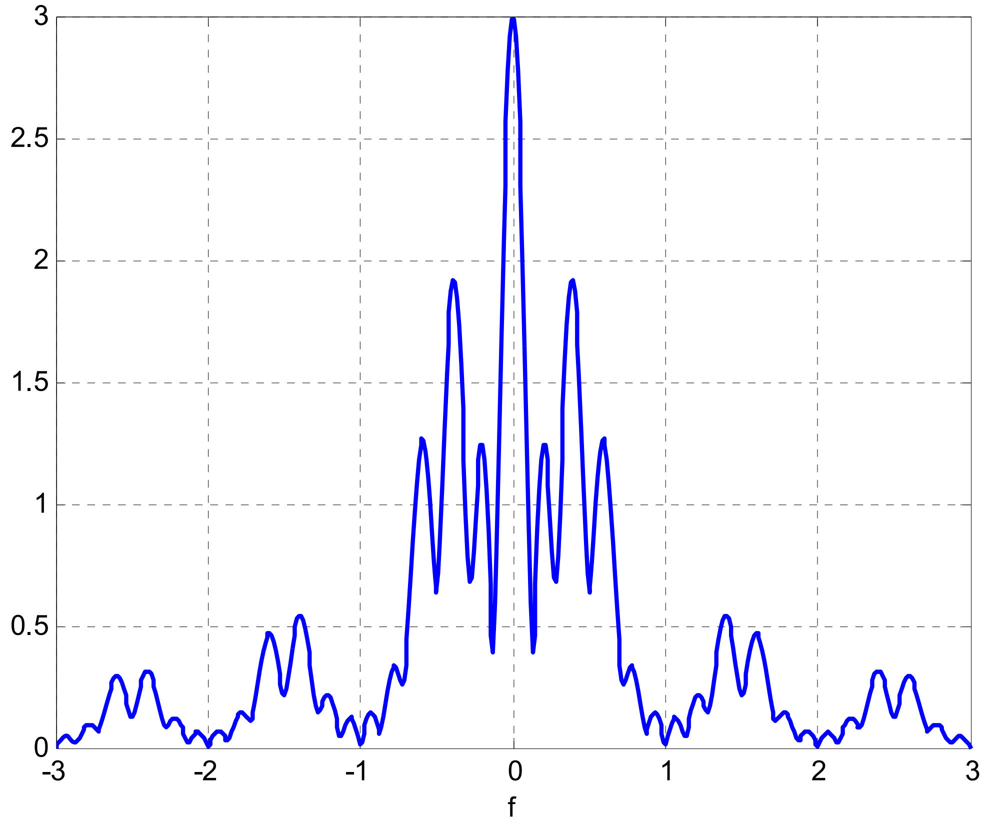


A revisit to an earlier OOK Example

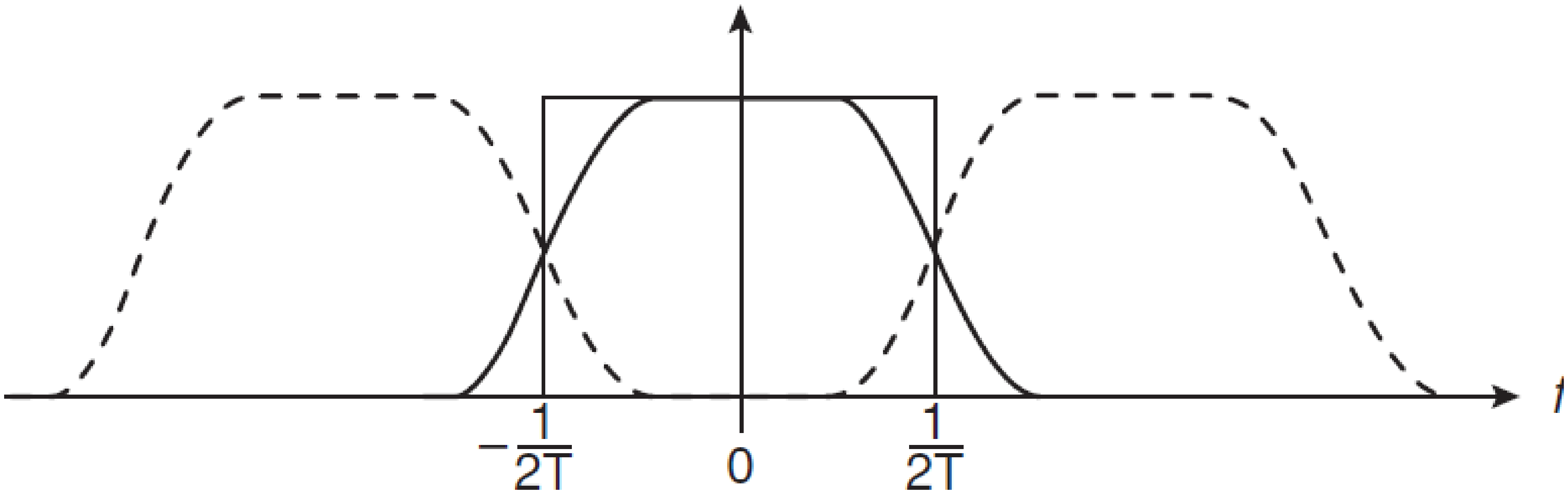
$$x_{\text{PAM}}(t) = \sum_n m[n]\, p(t - nT_s)$$



$\overset{\mathcal{F}}{\rightleftharpoons}$

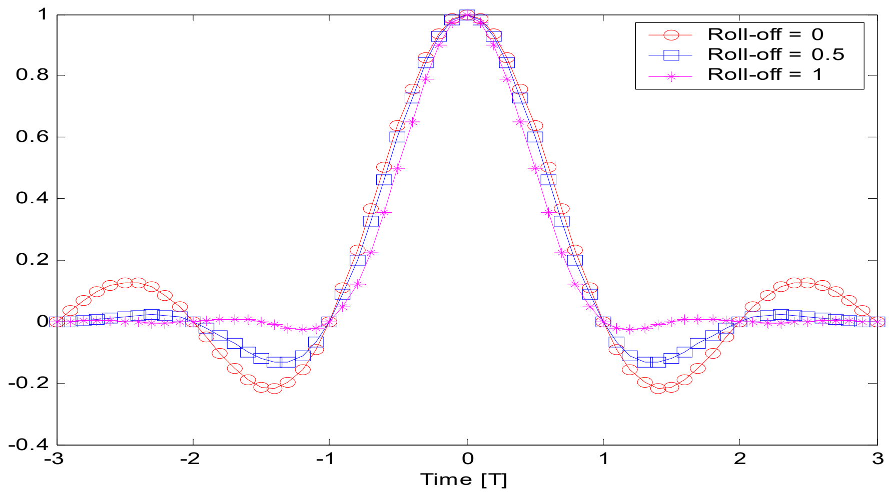


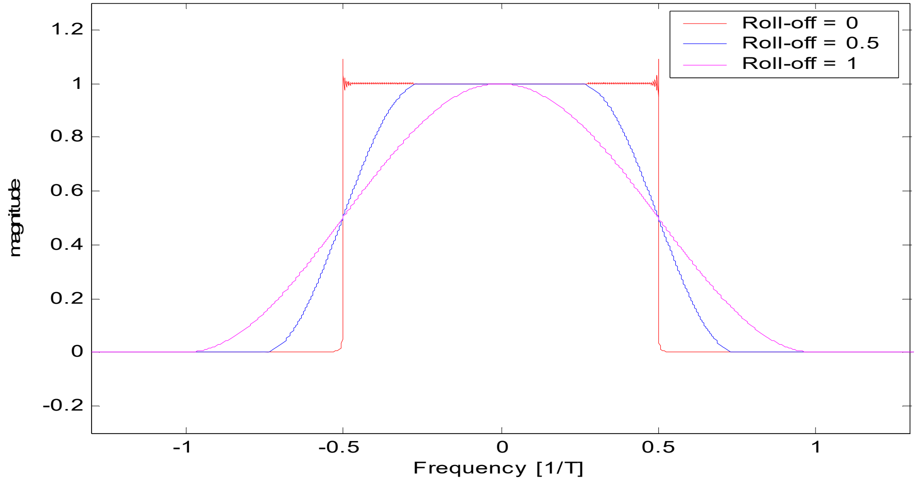
Nyquist criterion



[Blahut, 2008, Fig 2.9]

Raised Cosine Pulses





For fixed nonzero α, the tails decay as $1/t^3$ for large $|t|$.

Although the pulse tails persist for an infinite time, they are eventually small enough so they can be truncated with only negligible perturbations of the zero crossings.

$$p_{RC}(t;\alpha) = \frac{\cos\frac{\alpha\pi t}{T}}{1-\frac{4\alpha^2 t^2}{T^2}} \operatorname{sinc}\frac{\pi t}{T}$$

$$= \frac{\cos\frac{\alpha\pi t}{T}}{1-\frac{4\alpha^2 t^2}{T^2}} \frac{\sin\frac{\pi t}{T}}{\frac{\pi t}{T}}$$